Contributions of Muslim Mathematicians during the Golden Age of Islam

1) Dr. Aisha Ishrat Rizvi
Lecturer, Department of Islamic Learning, Faculty of Islamic Studies, University of Karachi, Karachi.
Draishaishrat@gmail.com
2) Dr. Alvina Afsar
Lecturer, Department of Islamic Learning, Faculty of Islamic Studies,
University of Karachi, Karachi.

KEYWORDS

Mathematics
Algebra
Geometry
Trigonometry
Number theory
Islamic civilization
Golden Era,
Al-Khwarizmi
Omar Khayyam
Al-Kindi
Al-Beruni
Al-Hasan

ABSTRACT

Math is not just a subject designed merely for one to spend time looking at problems and following some steps to arrive at a solution, rather it can be thought of as parent of all derived sciences – a language quantifying the very essence of nature and the universe. Many have described maths as the algorithm, the code with which God has written the universe. We see maths in our daily lives, with structures made using the theorems of geometry to flower patterns and movements of stars and galaxies explained by mathematical expressions. All of this could not have been possible without the almost uncountable advancements made by mankind in this field. Starting from ancient times in the early civilizations of Egypt and Mesopotamia, and later extending to Greek, Chinese and Indian, to this date, we witness every now and then some form of progress and development in maths. One such time period of development occurred in the Golden Era of Islamic civilization, which extended from the 7th to 14th century. During this time, many Muslim mathematicians arose to fame because of their extensive research and contribution to this field, with their work proving to be the best of their time and later on influencing the later civilizations and societies especially the West, to translate their work and build up theorems and laws upon them, which we now very conveniently use. Muslim mathematicians being influenced by their culture, were very interested in geometrical designs and made significant contributions in this sub-field. Perhaps one of the most famous, if not the most famous, sub-field in maths – algebra – was started by a Muslim mathematician named Al-Khwarizmi. Many other important noteworthy contributions were made by Muslim mathematicians in this subject ranging from trigonometry to number theory, with contributions made by Al-Khwarizmi, Omar Khayyam, Al-Kindi, Al-Beruni, Al-Hasan and the list goes on

1 Introduction

Brief history of maths

Muslim mathematicians took upon the works already done by the Greeks and Indians and extended them for a more generalized view. Some of them in a show of exceptional arithmetic and analytical skills built their own theorems and presented questions and solutions that were not thought of before (or at least not presented in the more complex and particular way that Muslim

*Corresponding Author Dr. Aisha Ishrat Rizvi
Email Address: Draishaishrat@gmail.com

mathematicians did). The most prevalent theories at that time, with their ownership belonging to different areas of the world – from the Greeks and Romans to the Indians and Chinese – were those of geometry and astronomy, number systems and physical mathematics – which sought to define natural phenomena and the order of nature through mathematical relationships, since inspiration to formulate those natural happenings was taken directly by the close admiration of nature and the conformity of its wonders and marvels to different patterns that implied a mathematical background. Some of the famous works already published by Greeks were Pythagoras' Theorem, Archimedes' approximation of Pi (π), Euclid's geometry, Diophantus' algebraic equations etc., and along with these famous Indian works as well such as Aryabhata's works on trigonometry, approximation of Pi (π), movement of cosmological bodies, Brahmagupta's work on zero and negative numbers, cyclic geometry, Varahamihira's work on trigonometry etc. Their works stood to be the best of the best in their own empires, and after their decline, those works were brought up by the now flourishing Muslim empire, which stretched from the present-day Spain and north-western Africa all the way to present day Iran and some parts of western Pakistan.

The islamic golden age

The Muslim empire started right after the end of the Rightly guided Caliphate (Khilafat-e Rashidun) – from Hazrat Abu Bakr (R.A.) to Hazrat Hassan (R.A.) – with the Umayyads being the first rulers and establishing a caliphate taking inspiration from their elders and ancestors (i.e. the Rightly guided Caliphs). After ruling for almost 90 years, the Umayyad Caliphate was replaced by the Abbasid Caliphate, which ruled the Islamic empire for a good 500 years. Although much expansion was done under the Umayyad Caliphate and the progress in science and maths and other fields took a rise, it was the Abbasid rule that the Islamic empire reached its peak, thus the period being known as the Golden Era of Islam. It was in this period, that Muslims showed a rapid development in worldly sciences as well as religious affairs. Baghdad, the capital of the empire, became the center of hub for learning and development and it was here that some of the greatest works in different fields were produced. The famous House of Wisdom (Bayt-al-Hikmah) or the Great Library of Baghdad, founded by the Abbasid Caliph Haroon-al-Rashid, housed many of the greatest scholars and experts of sciences alike, along with translators that belonged to all three big religions at that time i.e. Islam, Christianity and Judaism, with Christians and Jews being involved more in translating the works of Greeks and Indians and Persians to Arabic for better and more comprehensive understanding. During the reign of Caliph Ma'mun-al-Rashid, the translation progress reached new heights, and the library was made accessible to the public which furthered the pace of advancements. Although, at the end of the Abbasid Caliphate i.e. during the sack of Baghdad by Mongols, much of the work was lost forever, but that ones that survived proved to be much beneficial in the long run, with later empires and societies building their core works from the background laid by Muslims.

Reasons for starting progress

Though, through the hard efforts of the Caliphs and the general Muslims, the Islamic empire reached its peak, these advancements and progress was not done just to gain technological superiority to other empires at that time (this however, could be present as a very plausible reason), but the motives behind these swift expansions in sciences could be related to other reasons such as, due to rapid expansion of the Islamic empire, a need was sought to solve the challenges posed which mostly included calculating inheritance, regulating flow of money etc. Also, since the empire was expanding a fast rate, and many expeditions took place which mostly included travel by sea or through uncharted terrains, a challenge was always present to accurately predict (and calculate) positions, relative movements and time. Another factor that could have led to such speedy progress could be due to the fact that when the empire reached new areas that were previously under the rule of Greeks or Persians, they came across texts and papers and scriptures that showed their mathematical prowess even, and that fuelled the curiosity of Muslims to study them and extend their research and experimental findings. All of these (possible) reasons made it inevitable for Muslims to take up interest in this field and in the end these mathematicians produced some of the greatest early works. Another source of motivation to rigorously pursue this field can be taken directly from the Quran and Sunnah.

Allah says in the Quran,

"وَسَخَّرَ لَكُمُ اللَّيْلَ وَالنَّهَارَ وَالشَّمْسَ وَالْقَمَرَ وَالنُّجُومُ مُسَخَّرَاتٌ بِأَمْرِهِ ۗ إِنَّ فِي ذَٰلِكَ لَآيَاتٍ لِقَوْمٍ يَعْقِلُونَ"

"He has subjugated for you the day and the night and the sun and the moon, and the stars (too) are subservient by His command. Surely, in that, there are signs for a people who understand".

And also,

وَأَلْقَىٰ فِي ٱلْأَرْضِ رَوَٰسِيَ أَن تَمِيدَ بِكُمْ وَأَنْهَٰرًا وَسُبُلًا لَّعَلَّكُمْ تَهْتَدُونَ وَعَلَٰمَٰتٍ ۚ وَبِٱلنَّجْمِ هُمْ يَهْتَدُونَ

"He has placed into the earth firm mountains, so it does not shake with you, as well as rivers, and pathways so you may find your way. And He has set landmarks. And by the stars they find the right way".

In another part of the Quran, Allah says,

"آن رَبَّكُمُ ٱللَّهُ ٱلَّذِى خَلَقَ ٱلسَّمَٰوَٰتِ وَٱلْأَرْضَ فِى سِتَّةِ أَيَّامٍ ثُمَّ ٱسْتَوَىٰ عَلَى ٱلْعَرْشِ ۖ يُغْشِى ٱلَّيْلَ ٱلنَّهَارَ يَطْلُبُهُۥ حَثِيثًا وَٱلشَّمْسَ وَٱلْقَمَرَ وَٱلنُّجُومَ مُسَخَّرَٰتٍۭ بِأَمْرِهِۦٓ ۗ أَلَا لَهُ ٱلْخَلْقُ وَٱلْأَمْرُ ۗ تَبَارَكَ ٱللَّهُ رَبُّ ٱلْعَٰلَمِينَ"

"Surely, your Lord is Allah who created the heavens and the earth in six days, then He positioned himself on the Throne. He covers the day with the night that pursues it swiftly. (He created) the sun and the moon and the stars, subjugated to His command. Lo! To Him alone belong the creation and the command. Glorious is Allah, the Lord of all the worlds".

Moreover, Allah also says,

"وَهُوَ ٱلَّذِى جَعَلَ لَكُمُ ٱلنُّجُومَ لِتَهْتَدُوا۟ بِهَا فِى ظُلُمَٰتِ ٱلْبَرِّ وَٱلْبَحْرِ ۗ قَدْ فَصَّلْنَا ٱلْءَايَٰتِ لِقَوْمٍ يَعْلَمُونَ"

"And He is the one who made for you the stars, so that you may be guided by them in darknesses of the land and the sea. We have elaborated the signs for people who know".

From this, it is evident that Allah has made the universe to work on laws designed by Him, and it is through empirical observations and a closer and deeper understanding of these laws, that the Muslims are guided, both in their Deen (religious matters) and Duniya (worldly affairs). The pursuit of such knowledge is also mandated by the words of Prophet Muhammad ﷺ that goes,

حَدَّثَنَا عَلِيُّ بْنُ مُحَمَّدٍ، حَدَّثَنَا وَكِيعٌ، عَنْ أُسَامَةَ بْنِ زَيْدٍ، عَنْ مُحَمَّدِ بْنِ الْمُنْكَدِرِ، عَنْ جَابِرٍ، قَالَ قَالَ رَسُولُ اللَّهِ صلى الله عليه وسلم " "اسْأَلُوا اللَّهَ عِلْمًا نَافِعًا وَتَعْوِيذًا، وَذُوقُوا بِهِ مِنْ عِلْمٍ لَا يَنْفَعُ

"Ask Allah for beneficial knowledge and seek refuge with Allah from knowledge that is of no benefit".

And also,

وعنه قال: قال رسول الله صلى الله عليه وسلم: "من تعلم علمًا مما يبتغي به وجه الله عز وجل، لا يتعلمه إلا لصيب به غرضًا من الدنيا، لم يجد عرف الجنة يوم القيامة" (رواه أبو داود بإسناد صحيح)

"He who does not acquire knowledge with the sole intention of seeking the Pleasure of Allah but for worldly gain, will not smell the fragrance of Jannah on the Day of Resurrection".

In another Hadith, the Prophet Muhammad ﷺ said,

وعن أنس رضي الله عنه قال: قال رسول الله صلى الله عليه وسلم : "اللهم إنّي استفدتُ من علمٍ علمتنيه، وعلّمني ما ينفعني، وارزقني علمًا ينفعني".

Hazrat Anas (R.A.) narrated that the Messenger of Allah (ﷺ) used to say, "O Allah! Grant me benefit in what you have taught me and teach me useful knowledge and provide me with knowledge that will benefit me".

From this it is clear that, seeking beneficial knowledge is a duty upon Muslims, and while knowledge is mostly used in the context of religious knowledge in both Quran and Sunnah, pursuing worldly knowledge is also valuable given the intentions are pure, since pursuing and spreading that knowledge among others can result in ease of tasks that otherwise take more time and strength. Furthermore, as time proved in the future, pursuit of such (beneficial) knowledge lead to further advancements in the field of science and technology that proved useful to the Muslim Ummah based upon their era and time, as it helped in the spread of message of Islam by, for example, improvements in communications, better education for Muslim youth to keep them updated with both Deen and Duniya, strengthening of military powers etc. The strive of mathematical knowledge and skills by Muslims in that era was also based on these injunctions since the goal was to make the life of common Muslims and government (Khilafat) easy in everyday affairs such as regulating money and taxes, calculating inheritance and debt, etc., as also willed by Allah in the Quran. While these advancements did lead to worldly gains, it was the intentions that mattered and it was due to this that the Golden Era flourished for almost 8 centuries, and ultimately became a beacon of science and maths and other technological advancements. These advancements were done by many Muslim mathematicians pertaining to different centuries and their list is almost non-exhaustive. Nevertheless, some prominent mathematicians are discussed in this paper along with their respective works, under their respective sub-fields.

Works Of Muslim Mathematicians

Algebra

Perhaps the most notable contribution to maths by Muslim mathematicians was in the field of Algebra. This was primarily due to problems that arose in the calculation of debt and inheritance and other monetary and governance factors.

The most famous work done was by Muhammad Ibn Musa Al-Khwarizmi, or simply referred to as Al-Khwarizmi, who is also known as the "Father of Algebra". He was a polymath by profession, and had produced works in mathematics, astronomy ad geography. He made numerous contributions in maths besides algebra, which included trigonometry and number systems. His book "Al-Kitaab Al-Mukhtasar fi Hisaab Al-Jabr wal-Muqabalah" – The Compendious Book on Calculation by Completion and Balancing – is undoubtedly his most famous book, which outlined his work on algebra involving equations of 1st and 2nd order i.e. linear and quadratic, all written in a rhetorical manner (in words). It is full of examples and applications of this branch of mathematics providing a wide range of real-life problems that deal with trading, surveying and legal inheritance. It gives an account of how to solve such polynomial equations, by giving the methods of reduction and balancing, which today is conveniently known as cancellation of similar terms on both sides of the equation. AL Khwarizmi presented the methods to solve unknown variables in an elementary way to make it more generalized for all numbers (existing number system). He implied the use of his two operations namely Completion (Al-Jabr) that dealt with simplifying the equation, and Balancing (Al-Muqabalah) that sought to remove irrational and negative numbers, by adding or subtracting the same value from both sides

of the equation, a method now known as completing the square. Al-Khwarizmi's methods of solving equations involved relating unknown variables to physical objects and quantities. According to him, there are three (3) types of quantities: simple numbers such as 4, 9, 50, etc. which are known as whole numbers today; root numbers which he considered as unknown and called them "things" – denoted today by 'x'; and in the end, wealth or also known as mal, which is the square of the root of the unknown which is denoted today by x2. Furthermore, he stated six (6) basic types of equations
in his book which are:

1) Roots (things) equal numbers ($nnnn = mm$).
2) Wealth (squares) equal roots (things) ($nn2 = nnnn$).
3) Wealth (squares) equal numbers ($nn2 = mm$).
4) Numbers and wealth (squares) equal roots (things) ($mm + nn2 = nnnn$). 5) Numbers equal roots (things) and wealth (squares) ($mm = nnnn + nn2$). 6) Wealth (squares) equals numbers and roots (things) ($nn2 = mm + nnnn$).

An example of Al-Khwarizmi's approach to solve a quadratic equation is as follows, that would otherwise be solved very conveniently today using the quadratic formula or by graphing it. However, as mentioned above Al-Khwarizmi employed a procedure of cancellation and balancing. The first procedure Al-Khwarizmi uses in solving this problem is shown in Fig. 1, where he first halves the number of roots, where he receives 5. He then multiplies 5 by itself, where he receives 25. Next, he subtracts 21 from this product, where he receives 4. Further, he takes the square root of 4, where he obtains 2, and subtracts that from 5, where he then receives 3.

10 2
$-$ -21
2

[Fig. 1]

In his second procedure, he takes the exact same steps as in procedure 1, however, this time instead of taking half the roots and subtracting, he takes half the roots and adds this time. This yields the following expression, as shown in figure 2.

$5 + 52 - 21$

[Fig. 2]

The solution to figure 2 yields 7. In this procedure, he refers to 10 as "the number of roots", and 21 as the simple number.

Al-Khwarizmi describes the general solution of any quadratic equation of type 4 (as shown above), where n represents the number of roots and m represents any number as the following…

nn $nn\,2$
$\pm$ $-mm$
2 2

[Fig. 3]

He stated that there were no solutions whenever he received a number less than zero under the square root. Nowadays, we call these numbers imaginary. He also acknowledges that when the number under the square root is equal to zero, then only one solution exists. Also, whenever Al-Khwarizmi had a coefficient in front of $ppnn2$, he would divide by p, obtaining.

$2 + mm = nnnn$
nn
pp pp

This shows that his coefficients were not restricted to whole numbers only.

We will now turn to another example focusing on the fifth basic type of equation. In this example, we have 39 = $nn2 + 10nn$, where we have the number equals roots and wealth. AlKhwarizmi uses an algebraic proof and a geometric proof. We will first look at the algebraic proof which is as follows: The first step is to take half of the roots, 10, which gives us 5. We then multiply it by itself, which is 25. We then add this to 39, where we receive 64. We take the square root of 64, which is 8 and subtract it from it half the roots, 5, which leaves us with 3, our solution.

Al-Khwarizmi also implied a geometrical prove to provide a solution for the polynomial equations, which is now known as completing the square method.

Next, we will take a look at his geometric proof. In the first step, Al-Khwarizmi starts with a square, where each side length is represented by x. Therefore, the area of the square is $nn2$ (figure 4). Now that we have $nn2$, we must now add $10nn$. We do this by adding four rectangles, each or in length and length x to the square. Here we now have $nn2 + 10nn$, which in our example equals to 39 (figure 4). Last, Al-Khwarizmi finds the area of the four little squares, which is × which gives us . Thus, the outside square of figure 4 has an area of × 4 + 39 since the area of the 4 squares are and we have the $nn2 + 10nn$ left which we already know is equal to 39. Solving for the area, we receive 25 + 39, which equals 64. Therefore, the side length of the square is 8, since the square root of 64 is 8. The side length is equal to + nn + . This can be seen from figure 4 where the two squares have a side length of . Therefore, nn + 5 = 8, so nn = 3. This technique works because once we find the area of the square above, we can use that to determine what the x-value would equal by determining its square root.

Al-Khwārizmī also provides a second way to "complete the square," which might be easier for modern algebra students to use. Using the equation: x2 + 10x = 39, he creates the following figure. First, we make the square with sides equal to x.

Again, the area of this square is x2 = 39 – 10x. Then, he added on rectangles with lengths 10/2 (= 5). So, the area of this figure (the square and two rectangles) is equal to 39. In other words: x2 + 2(5x) = x2 + 10x = 39. He then completed the figure to make a large square by finishing the square with sides of length equal to 5, which gave a figure that can be used to find x.

Then, al-Khwārizmī uses the same method he used above to get the area of the completed large square, with sides equal to x + 5: (x2 + 10x) + 52 = 39 + 25 = (x+5)2 = 64

And he gets the same result as he did above: (x+5) = 8, -8, thus bearing the answer: x = 3, -13.

Another famous Muslim mathematician to produce influential works on algebra was by Abu Kamil Shuja Ibn Aslam Ibn Muhammad Ibn Shuja. He was known as Al-Haasib Al-Misri or "The Egyptian Calculator", and is considered as the successor to Al-Khwarizmi. He made important contributions such as systematically using irrational numbers as solutions and coefficients in algebraic equations. He involved powers higher than 2nd degree up to 8th degree. Like Al-Khwarizmi, he too wrote books in a rhetorical manner. Along with this, he illustrated the rules of signs for expanding multiplication such as in (a ± b) (c ± d). He wrote a few books which outlined his works, which are interconnected in their terms of content. His book Kitaab Al-Jabr Wal-Muqabalah – Book of Algebra – is his most influential work, which is an expansion of the work done on algebra by Al-Khwarizmi. In this book, he solves systems of equations whose solutions are whole numbers and fractions, along with the acceptance of irrational numbers as both coefficients and solutions to quadractic equations. The book illustrates how the problems apply to geometry and how irrationalities can be used to solve problems pertaining to polygons. It also contains solutions of some indeterminate equations and problems of application of such equations in both realistic and hypothetical situations. In his another book, Kitaab Al-Tara'if Fi'l Hisaab – Book of Rare Things in the Art of Calculation – he describes a series of systematic procedures to find integral solutions for indeterminate equations. In his book Kitaab Al-Mukhammas Wal-Mu'ashshar – Book on the Pentagon and Decagon – he discusses algebraic methods to solve geometrical problems, such as using a quartic equation (x4 + 3125 = 125x2) to calculate a numerical approximation for the side of a pentagon inside a circle. He also made use of the Golden Ratio in some of his calculations. In his book Kitaab Al-Tair – Book of Birds – Abu Kamil outlines methods to solve indeterminate linear equations whose solutions were of the positive integral form. Lastly, in his book Kitaab Al-Misaha Wal-Handasa – Book on Measurement and Geometry – Abu Kamil provides sets of rules for calculating the volume and surface area of solids such as prisms, square pyramids, circular cones, parallelepiped, etc. with the book serving as a manual for non-mathematicians. One of his lost books Kitaab Al-Wasaya Bi Al-Jabr Wal-Muqabalah – Book of Estate Sharing using Algebra – contained algebraic solutions of problems pertaining to inheritance, which certainly would have been of immense importance in that time. His other work included his book on false position which is discussed later.

Abu Bakr Muhammad Ibn Al Hassan Al-Karaji was another prominent Muslim mathematician who made important contributions to algebra. He wrote a few books of which surviving and the most famous are Kitaab Al-Badl Fi'l-Hisaab (Book of Wonderful on Calculation), Kitaab

Al-Fakhri Fi'l-Jabr Wal-Muqabalah (Book of Glorious on Algebra) and Kitaab Al-Kafi Fi'l Hisaab (Book of Sufficient on Calculation). He provided a systematic study of algebra of exponents and was the first to define rules of powers of 'x', and their reciprocals in the case of multiplication and division. He too, used words instead of mathematical notation to denote these powers. He extended previous works by providing rules for arithmetic operations on polynomials. Perhaps, his most famous work was on binomial coefficients and the binomial theorem, for which he is regarded as the first mathematician for its discovery. Using this work, he also gave a first definition of Pascal's Triangle. He is also credited for the introduction of the idea of argument by mathematical induction with which he found the general formula for the sum of integral cubes.

Another influential Muslim mathematician was Abu Al-Hasan Ibn Zahrun Al-Harrani Al-Sabi, or also known as Thabit Ibn Qurra. He followed Al-Khwarizmi's work and presented his findings and demonstrations of proofs and solutions of equations in a general way. He found a formula for amicable numbers, which are pairs of numbers that have a special relationship. He showed how to find these numbers easily in his book called "Treatise on the Derivation of the Amicable Numbers in an Easy Way", using number theory and geometry. Thabit also worked on problems involving chessboards using geometrical relations and exponential series. He used combinatorics and permutations to provide possible number of solutions to win a game.

Sharaf Al-Din Al-Muzaffar Ibn Muhammad Ibn Al-Muzaffar Al-Tusi, or simply known as Sharf Al-Din Al-Tusi, was another notable Muslim mathematician in the Golden Age. He was an Iranian mathematician and astronomer. He is attributed with first putting forward the concept of a function however, he did not make it very clear. It took another 500 years for Leibniz, a German scholar, to develop the idea of a dynamic function. Sharaf al-Din also found a way to estimate the root of a cubic equation numerically, using what is now known as the "Ruffini-Horner method". He also came up with a new method to determine how many solutions (two, one, or none) a certain kind of cubic equation would have, depending on the value of a constant term. Al-Tusi only considered positive solutions, because negative and zero solutions were not yet accepted as valid. The cubic equations he studied had the form $f(x) = c$, where $f(x)$ is a cubic polynomial with a negative coefficient for the x3 term, and c is positive. He divided these equations into five types, based on the signs of the other coefficients of f(x). For each type, he found the value of m where the function f(x) reached its maximum, and proved geometrically that f(x) was always less than f(m) for any positive x that was not equal to m. He then concluded that the equation had two solutions if c was less than f(m), one solution if c was equal to f(m), or no solutions if f(m) was less than c. Though Al-Tusi's findings were nothing short of extraordinary for his time, he however, did not explain how he found the values of m that gave the maxima of the functions f(x), which have divided scholars on whether Al Tusi could have done this by "systematically" finding the derivative of the function f(x), and setting it to zero, or not. The difference between f(m) and c, which is denoted by D, is called the discriminant of the cubic polynomial obtained by subtracting one side of the original cubic equation from the other – this was obtained from Al-Tusi's conditions for the number of roots of the cubic equations. Sharaf al-Din studied the equation x3 + d = b x2 by rewriting it as x2 (b - x) = d and saying that the left-hand side had to be at least as big as d for the equation to have a solution. He then found the maximum value of the left-hand side. If this value was smaller than d, there was no positive solution; if it was equal to d, there was one solution; and if it was bigger than d, there were two solutions. Sharaf al-Din's work on this equation was an important advance in Islamic mathematics, but it was not followed up by anyone else in the Muslim world or in Europe. He also wrote a book called "Treatise on Equations" which has been hailed by some as the beginning of algebraic geometry.

Another big name in the Golden Age was of Ghiyath Al-Din Abu Al-Fath Umar Ibn Ibrahim Nisaburi, or commonly known as Omar Khayyam. He was a polymath by profession and made important contributions to cubic equations and conics. Some of his surviving work include Commentary on the Difficulties Concerning the Postulates of Euclid's Elements (Risāla fī Sharḥ mā Ashkal min Muṣādarāt Kitāb Uqlīdis), (ii) Treatise On the Division of a Quadrant of a Circle (Risālah fī Qismah Rub'al-Dā'irah), and (iii) Treatise on Algebra (Risālah fi al-Jabr wa'l-Muqābala). Undoubtedly, his most famous work is that on the solution of cubic equations. Khayyam was the first to develop a general theory of cubic equations, and the first to find geometric solutions for all kinds of cubic equations, as long as they had positive roots. His work on cubic equations is in his Treatise on Algebra. Khayyam listed all the possible equations that involve lines, squares, and cubes. He had three binomial equations, nine trinomial equations, and seven tetranomial equations. He gave numerical solutions for the first- and second-degree polynomials by constructing them geometrically. He found that there are fourteen different types of cubics that cannot be simplified to a lower degree equation. For these, he could not construct his unknown segment with a ruler and a compass. He used the properties of conic sections to give geometric solutions for all types of cubic equations. He needed some lemmas from Euclid and Apollonius for his geometric proof. The positive root of a cubic equation was found as the x-coordinate of a point where two conics meet i.e. the abscissa, for example, where two parabolas cross, or where a parabola and a circle meet, etc. However, he admitted that he did not know how to solve these cubics with arithmetic and said that "maybe someone else will know it after us". This problem was not solved until the sixteenth century, when Cardano, Del Ferro, and Tartaglia in Italy found the algebraic solution of the cubic equation in general. Khayyam's work is an attempt to combine algebra and geometry. His geometric method of solving cubic equations was further studied by M. Hachtroudi and used to solve fourth-degree equations. Although some similar methods had been used before by Menaechmus and Abu al-Jud, Khayyam's work is the first systematic and exact method of solving cubic equations. Omar Khayyam is also believed to have known the general formula for binomial expansion due to his ability to extract roots of equations.

Apart from these mathematicians, there were others too who made contributions to algebra. Al-Samaw'al Ibn Yahya Al-Maghribi, a Muslim convert, wrote a treatise called Al-Bahir Fi'l Jabr – The Brilliant in Algebra. He gave rules for signs creating the concepts of positive (excess) and negative (deficient) numbers. He also gave rules for subtraction of powers, along with methods to do multiplication and division on simple expressions that involved fractions. Furthermore, he also gave examples of the division of complex polynomials. He used the basic concept of mathematical induction to extend the binomial theorem upto n=12, as well as Pascal's Triangle. Some work was also done by Hasan Ibn Al-Haytham, who extended the work of Euclid and Thabit Ibn Qurra to create a link between algebra and geometry. In doing so, he developed a formula for summing up the first 100 natural numbers and proving his formula using geometric proofs.

Geometry

Muslim mathematicians made significant contributions to the field of geometry with works involving mostly shapes and patterns, reminders of which can be seen throughout the world in buildings of that time, masjids, and other structures. Advancements in this field were made to provide proofs for equations, as well as to find directions such as the case in navigation and positions of stars and other celestial bodies for direction of the Qibla (Ka'aba).

One of these mathematicians who pioneered in geometry was Omar Khayyam. He wrote a treatise in which he made an attempt to solve Euclid's 5th Postulate. Khayyam was the pioneer in examining the three unique cases of acute, obtuse, and right angles in a Khayyam-Saccheri quadrilateral. He established several theorems about these angles, demonstrated that the right-angle hypothesis leads to Postulate V, and dismissed the obtuse and acute cases as inconsistent. His comprehensive effort to validate the parallel postulate significantly influenced the progression of geometry, as it highlighted the potential for non-Euclidean geometries. Today, we understand that the hypotheses of acute, obtuse, and right angles correspond to the non-Euclidean hyperbolic geometry of Gauss-Bolyai-Lobachevsky, Riemannian geometry, and Euclidean geometry, respectively. Furthermore, in his one treatise, he applied geometry to algebra, devoting himself in the investigation of the possibility of division of a circular quadrant into two parts with their line segments from the dividing point and the perpendicular diameters of the circle resulting in specific ratios. This led him to construct several curves which in turn lead to polynomial equations.

Thabit Ibn Qurra also made some significant contributions to geometry. Thābit presented an expanded proof of the Pythagorean Theorem, which incorporated an understanding of Euclid's fifth postulate. This postulate asserts that two intersecting straight lines form two interior angles that total less than 180 degrees. Thābit's use of reduction and composition techniques led to a synthesis and expansion of existing and historical knowledge on this renowned proof. He held the view that geometry was associated with the equality and differences in the magnitudes of lines and angles, and that concepts of motion (and broader physics concepts) should be incorporated into geometry. Beyond Thābit's contributions to Euclidean geometry, there are indications that he was well-versed in Archimedes' geometry as well. His work with conic sections and the computation of a paraboloid shape (cupola) demonstrates his skill as an Archimedean geometer. This is further highlighted by Thābit's application of the Archimedean property to develop a basic approximation of a paraboloid's volume. His use of uneven sections, while relatively straightforward, indicates a deep understanding of both Euclidean and Archimedean geometry. Thābit also authored a commentary on Archimedes' Liber Assumpta.

Another mathematician to make important contributions to this field was Ibn Al-Haytham. Ibn Al-Haytham delved into what is currently recognized as the Euclidean parallel postulate, the fifth postulate in Euclid's Elements. He employed a proof by contradiction, effectively incorporating the notion of motion into geometry. He developed the Lambert quadrilateral, which Boris Abramovich Rozenfeld referred to as the "Ibn al-Haytham–Lambert quadrilateral". However, his work was criticized by Omar Khayyam, who noted that Aristotle had previously denounced the use of motion in geometry. In the realm of basic geometry, Ibn Al-Haytham endeavoured to square the circle using the area of lunes (moon-like shapes), but eventually abandoned this impossible task. The pair of lunes created from a right triangle by constructing a semicircle on each side of the triangle, inward for the hypotenuse and outward for the other two sides, are referred to as the lunes of Al-Haytham. Interestingly, these lunes have the same combined area as the triangle itself. Al-Haytham also formulated his problem which comprised of drawing lines from two points in a plane of a circle converging at a point on the circumference and making equal angles with the normal at that point. He used his results on sums of integral powers that allowed him to calculate the volume of a paraboloid, and eventually used conic sections and geometric proof to solve the problem.

Another reputed personality in this era was Abu Sahl Wayjan Ibn Rustam Al-Kuhi, or simply Abu Sahl Al-Quhi, who is considered one of the best geometers of his time. He authored a scholarly work on the astrolabe, in which he tackled a series of complex geometric problems. In the field of mathematics, he focused on Archimedean and Apollonian problems that led to equations beyond the second degree. He managed to solve some of these problems and explored the conditions for their solvability. For instance, he successfully solved the problem of inscribing an equilateral pentagon into a square, which resulted in a fourth-degree equation. He also penned a treatise on the "perfect compass", a compass with an adjustable leg length that enables users to draw any conic section, including straight lines, circles, ellipses, parabolas, and hyperbolas. It is believed that al-Qūhī was the inventor of this device.

Some other contributions were made by Ibrahim Ibn Sinan, a notable figure in the field of geometry, who particularly focused on the study of tangents to circles. He made significant progress in the quadrature of the parabola and the theory of integration, expanding upon the work of Archimedes, which was not accessible during his time. Owing to his contributions, Ibrahim ibn Sinan is often regarded as one of the most influential mathematicians of his era.

Muslim mathematicians provided the world with some intricate geometric designs and the maths (geometry) behind it that were incorporated in buildings and structures by later Islamic empires and other societies. Some of these designs are as follows:

Trigonometry

The reason for the emergence of trigonometry was astronomy, which Muslims diligently studied, especially because of its significance for determining the exact time of Prayers and to determine the position of Qibla. Before the Muslims, Greek astronomers used to calculate the sides and angles of certain triangles with indicators of known sides and angles in order to understand the movement of the Sun, the Moon, and the five planets known at that time.

Muslim scientists have made a great contribution to the development of trigonometry, in particular spherical. Their interest in this field was determined by the problems of astronomy and geodesy, the main of which were:

- Accurate determination of the time of day.
- Calculation of the future location of the heavenly bodies, the moments of Sunrise and Sunset, eclipses of the Sun and Moon.
- Finding the geographical coordinates of the current location.

• Calculating the distance between cities with known geographical coordinates; determining the direction to Mecca (Qibla) from a given location.

Since it closely involved geometry and also because Muslims were fond of patterns and structures, some valuable works were produced throughout the medieval Islamic era. In order to observe holy days on the Islamic calendar in which timings were determined by phases of the moon, astronomers initially used Menelaus' method to calculate the place of the moon and stars, though this method proved to be clumsy and difficult. It involved setting up two intersecting right triangles; by applying Menelaus' theorem it was possible to solve one of the six sides, but only if the other five sides were known. To tell the time from the sun's altitude, for instance, repeated applications of Menelaus' theorem were required. For medieval Islamic astronomers, there was an obvious challenge to find a simpler trigonometric method.

In the early 9th century AD, Muhammad ibn Musa al-Khwarizmi produced accurate sine and cosine tables, and the first table of tangents. He was also a pioneer in spherical trigonometry on which he also wrote a treatise. He also authored some papers such as one on determination of direction of Makkah, and books incorporating spherical trigonometry and astronomy to build sundials and astrolabes. In 830 AD, Habash al-Hasib al-Marwazi produced the first table of cotangents.

By the 10th century AD, in the work of Abū al-Wafā' al-Būzjānī, all six trigonometric functions were used. Abu al-Wafa had sine tables in 0.25° increments, to 8 decimal places of accuracy, and accurate tables of tangent values. He also developed the following trigonometric formula: Sin2x =2sinx.cosx

In his original text, Abū al-Wafā' states: "If we want that, we multiply the given sine by the cosine minutes, and the result is half the sine of the double". Abū al-Wafā also established the angle addition and difference identities presented with complete proofs:
Very abundantly used in mathematics and engineering nowadays.

For the second one, the text states: "We multiply the sine of each of the two arcs by the cosine of the other minutes. If we want the sine of the sum, we add the products, if we want the sine of the difference, we take their difference".

He also discovered the law of sines for spherical trigonometry i.e.:

Also, in the late 10th and early 11th centuries AD, the Egyptian astronomer Ibn Yunus performed many careful trigonometric calculations and demonstrated the following trigonometric identity.

Another prominent mathematician in this field was Abū ʿAbd Allāh Muḥammad ibn Jābir ibn Sinān al-Raqqī al-Ḥarrānī aṣ-Ṣābiʾ al-Battāni, or simply known as Al-Battani. Al-Battani made significant strides in the field of geometry, particularly in the use of sines and tangents for calculations, superseding Ptolemy's geometrical methods. His work involved intricate mathematics, and he recognized the advantages of trigonometry over geometrical chords. He also understood the relationship between the sides and angles of a spherical triangle, now given by the formula: $\cos \alpha\alpha = \cos bb \cos cc + \sin bb \sin cc \cos AA$

He also produced a number of trigonometrical relationships such as

$\sin \alpha\alpha \tan \alpha\alpha = \cos \alpha\alpha$

$\sec \alpha\alpha = 1 + (\tan \alpha\alpha)2$

He also solved the equation: $\sin nn = yy \cos nn$, discovering the formula:

yy

$\sin nn =$

$1 + yy2$

He expanded on the concept of tangents, originally proposed by Iranian astronomer Habash alHasib al-Marwazi, to formulate equations for calculating tangents and cotangents. Al-Battani discovered their reciprocal functions, secant and cosecant, and compiled the first table of cosecants, referred to as a “table of shadows”, containing table of cosecants for each degree from 1 to 90, in relation to the shadow cast on a sundial.

Al-Battani utilized these trigonometric relationships to devise an equation for determining the qibla, the direction Muslims face during their daily prayers. However, his equation did not account for the Earth’s spherical shape, resulting in inaccurate directions. Despite this, it was widely used, particularly for those near Mecca. Al-Biruni, a polymath, later introduced more accurate methods, superseding Al-Battani’s equation, which was given by:

$\sin\Delta\lambda$

$\tan q =$, where $\Delta\lambda$ is the difference between longitude of the place and Mecca, and $\Delta\phi$ is the difference between the latitude of the place and Mecca.

Al-Battani is also known for his work “Tajrīd uṣūl tarkīb al-juyūb” (“Summary of the principles for establishing sines”), a concise treatise on trigonometry.

In the 15th century, Jamshīd al-Kāshī provided the first explicit statement of the law of cosines in a form suitable for triangulation. In France, the law of cosines is still referred to as the theorem of Al-Kashi. He also gave trigonometric tables of values of the sine function to four sexagesimal digits (equivalent to 8 decimal places) for each 1° of argument with differences to be added for each 1/60 of 1°.

One other prominent mathematician to take up this field was Muhammad ibn Muhammad ibn al-Hasan al-Tusi, or simply known as Nasir Al-Din Al-Tusi. Tusi is widely regarded as one of the greatest scientists of medieval Islam, since he is often considered the creator of trigonometry as a mathematical discipline in its own right. Al-Tusi was the first to pen a treatise on the subject separate from astronomy. His work, Treatise on the Quadrilateral, provided a comprehensive overview of spherical trigonometry as a distinct field, marking the first-time trigonometry was recognized as an independent branch of pure mathematics, separate from its long-standing association with astronomy.

Al-Tusi was also the first to identify the six unique cases of a right triangle in spherical trigonometry, building upon the work of earlier Greek mathematicians like Menelaus of Alexandria, who authored Sphaerica, a book on spherical trigonometry, and earlier Muslim mathematicians such as Abū al-Wafā’ al-Būzjānī and Al-Jayyani.

In his work On the Sector Figure, Al-Tusi presented the renowned Sine Law for plane triangles. He also introduced the sine law for spherical triangles, discovered the law of tangents for spherical triangles, and provided proofs for these laws.

$$\frac{\alpha}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Other works in trigonometry by Muslim mathematicians included those done by Abu Nasri Mansur ibn Ali ibn Iraq al-Jaʿdī, or simply known as Abu Nasr Mansur who is known for his work on spherical sine law. He also developed some of his work from the writings of Ptolemy, whilst also preserving writings of other Greek mathematicians. Abu Rayhan Muhammad ibn Ahmad al-Biruni, or simply Al-Biruni, also made some contributions in this field specifically in his works on astronomy and geodesy.

Number Theory and Arithmetic

Muslim mathematicians made several advancements in the number systems that existed at that time, mainly taking inspiration from the Indian/Hindi numerical system, and used their joint findings to develop and expand their works. Number theory interested the Greeks and they studied special kinds of whole numbers (even, odd, squares, etc.) This interest continued to the end of the ancient period. Euclid thought up “perfect numbers”, which are the sum of their proper divisors, an example being 6=3+2+1, where 3, 2 and 1 are divisors. Therefore, ideas around the study of numbers had a long history in the ancient world.

The explorations in fields of Arabic mathematics started in number theory and mathematicians made great contributions building on the studies found in earlier mathematical traditions. It has been proved that the first great contribution to number theory, in particular, regarding amicable numbers, is credited to mathematicians of Arabic science.

The beginning of Arab contribution in number theory emerged with the mathematician Thabit

Ibn Qurra and the theorem he expounded in Kitab al-a’dad al-mutahabba (Book of Amicable Numbers). Other mathematicians, such as Kamal al-Din Al-Farisi and Muhammad Baqir Yazdi, contributed to number theory and they obtained their results by using Thabit’s theorem. Historians showed that the study of number theory formed a continuous tradition and led to the discovery of theorems or problems usually ascribed to Western mathematicians several centuries later. For example, the appearance of Wilson’s theorem in the work of Ibn AlHaytham, Bechet’s problem of the weights in Al-Khazini, or the summation of the fourth powers of the integers 1, 2,… n in the work of 10th-century mathematician Abu Saqr al-Qabisi. Although he is more widely known for his work in medicine, Ibn Sina (or Avicenna, as he is known in Europe), also provided some work on number theory.

Al-Khwārizmī, a notable figure in the field of arithmetic, produced two significant works. The first, “Kitāb al-ḥisāb al-hindī” (Book of Indian computation), and possibly a simpler text, “Kitab al-jam’ wa’l-tafriq al-ḥisāb al-hindī” (Addition and subtraction in Indian arithmetic), are known to us only through Latin translations. These works outlined algorithms for decimal numbers (Hindu–Arabic numerals) that could be executed on a dust board, or takht (Latin: tabula), a tool used for calculations where figures could be written with a stylus and easily erased or replaced. Al-Khwarizmi’s methods were in use for nearly 300 years before being superseded by Al-Uqlidisi’s pen-and-paper algorithms. These texts were part of a 12th-century influx of Arabic science into Europe through translations and had a revolutionary impact. AlKhwarizmi’s Latinized name, Algorismus, became synonymous with the computational method he described, giving rise to the modern term “algorithm”. This method gradually replaced the abacus-based techniques previously used in Europe. Al-Khwarizmi’s contributions to arithmetic played a crucial role in introducing Arabic numerals, which were based on the Hindu–Arabic numeral system developed in Indian mathematics, to the Western world. The term “algorithm” originates from the algorism, the arithmetic technique with Hindu-Arabic numerals developed by al-Khwārizmī. Both “algorithm” and “algorism” are derived from the Latinized versions of al-Khwārizmī’s name, Algoritmi and Algorismi, respectively.

In the 9th century, Islamic mathematicians were familiar with negative numbers from the works of Indian mathematicians, but the recognition and use of negative numbers during this period remained timid. Al-Khwarizmi did not use negative numbers or negative coefficients. But within fifty years, Abu Kamil illustrated the rules of signs for expanding the multiplication. AlKaraji wrote in his book al-Fakhrī that "negative quantities must be counted as terms". In the

10th century, Abū al-Wafā' al-Būzjānī considered debts as negative numbers in A Book on What Is Necessary from the Science of Arithmetic for Scribes and Businessmen.

By the 12th century, al-Karaji's successors were to state the general rules of signs and use them to solve polynomial divisions. As al-Samaw'al writes:

"The product of a negative number — al-nāqiṣ — by a positive number — al-zāʾid — is negative, and by a negative number is positive. If we subtract a negative number from a higher negative number, the remainder is their negative difference. The difference remains positive if we subtract a negative number from a lower negative number. If we subtract a negative number from a positive number, the remainder is their positive sum. If we subtract a positive number from an empty power (martaba khāliyya), the remainder is the same negative, and if we subtract a negative number from an empty power, the remainder is the same positive number".

Islamic mathematicians including Abū Kāmil Shujāʿ ibn Aslam and Ibn Tahir alBaghdadi slowly removed the distinction between magnitude and number, allowing irrational quantities to appear as coefficients in equations and to be solutions of algebraic equations. Let us start with Ibn Sina and some of his works on the number theory. His important work entitled Alai in Persian and Kitab Al-Shifa in Arabic (Book of Healing), contains sections on arithmetic. He began a discussion, based on Greek and Indian sources, of different types of numbers (e.g. odd, even, deficient, perfect and abundant numbers) and explained different arithmetical operations, including the rule for 'casting out nines'. Examples from this work are the explanations:

- 6 is a perfect number since the sum of its proper divisors is 1+2+3=6,
- 8 is deficient number since the sum of its proper divisors is 1+2+4<8,
- 12 is abundant number since the sum of its proper divisors is 1+2+3+4+6>12, The rule of casting out nines is:

The sum of digits of any natural number when divided by 9 produces the same remainder as when the number itself is divided by 9. For example,

1- Add the digits of the number 436 to get 13, whose digits are then added to get 4 (the remainder when divide the number by 9),

2- Add the digits of the number 659 to get 20, whose digits are then added to get 2 (the remainder when divide the number by 9),

3- The product of the two numbers (436 and 659) is 287324, add the digits to get 26, whose digits are then added to get 8 (the remainder when divide the number by 9),

So casting out nines leaves remainder of 4, 2 and 8 respectively, and since 4×2=8, the multiplication is probably correct.

Ibn Al-Haytham also made some contributions on number theory which included his works on perfect numbers. In his work, Analysis and Synthesis, it's believed that he was the first to propose that every even perfect number can be expressed as (2n-1(2n − 1)), where 2n − 1 is a prime number. However, he couldn't prove this theory. It was later proven by Euler in the 18th century and is now known as the Euclid–Euler theorem. Al-Haytham, on the other hand, tackled congruence problems using what is presently referred to as Wilson's theorem. In his work, Opuscula, Ibn Al-Haytham examined the resolution of a system of congruencies and proposed two general solution methods. The first method, known as the canonical method, utilized Wilson's theorem. The second method employed a variant of the Chinese remainder theorem. He also wrote a treatise entitled "Finding the Direction of Qibla by Calculation" in which he discussed finding the Qibla, where prayers (salat) are directed towards, mathematically.

Other prominent work in this field was Omar Khayyam who worked on the real number concept. In his analysis of Euclid, Khayyam made significant contributions to the theory of proportions and the compounding of ratios. He explored the connection between the concepts of ratio and number, and explicitly pointed out various theoretical challenges. Specifically, he advanced the theoretical study of the concept of an irrational number. Unhappy with Euclid's definition of equal ratios, Khayyam redefined the concept of a number using a continuous fraction to express a ratio. Youschkevitch and Rosenfeld suggest that Khayyam's approach of treating irrational quantities and numbers on the same operational scale initiated a significant shift in the doctrine of number. Similarly, D. J. Struik noted that Khayyam was paving the way towards the extension of the number concept that leads to the notion of the real number.

Abū Yūsuf Yaʿqūb ibn ʾIsḥāq aṣ-Ṣabbāḥ al-Kindī, or simply known as Al-Kindi, also made significant progress in this domain. al-Kindi played an important role in introducing Hindu numerals to the Islamic world, and their further development into Arabic numerals along with al-Khwarizmi which eventually was adopted by the rest of the world. Al-Kindi was the author of numerous significant mathematical works, covering topics such as arithmetic, geometry, Hindu numbers, numerical harmony, lines and number multiplication, relative quantities, measurement of proportion and time, numerical procedures, and cancellation. He also penned a four-volume work titled "On the Use of the Hindu Numerals" (Kitāb fī Istimāl al-'Adād al-Hindīyyah), which played a crucial role in spreading the Hindu system of numeration in the Middle-East and the West. In the field of geometry, he wrote on the theory of parallels among other topics. He also composed two works related to geometry on optics. As a philosopher, he utilized mathematics to challenge the concept of the eternity of the world, arguing that actual infinity is a mathematical and logical contradiction.

Abu'l Hasan Ahmad ibn Ibrahim Al-Uqlidisi, commonly known as Al-Uqlidisi, was another notable mathematician to produce works in this domain. He is credited with the earliest known book that introduces the positional use of Arabic numerals, titled "Kitab al-Fusul fi al-Hisab al-Hindi" (The Arithmetics of India), which was written around 952. The book is particularly remarkable for its exploration of decimal fractions along with signs, and its demonstration of how to perform calculations without erasing.

Some other works in this field included those presented by Abu-Abdullah Muhammad ibn Īsa Māhānī. Most of his work revolved around geometry, arithmetic and algebra. He wrote commentaries on earlier works – of Greek mathematicians. In one of his commentary, he worked on the concept of ratios, proposing a theory on the definintion of ratios based on continued fraction. In one of his commentary, he delved into the study of irrational numbers, including those of quadratic and cubic nature. He extended Euclid's definition of magnitudes— which was originally limited to geometric lines—by incorporating integers and fractions as rational magnitudes, and square and cubic roots as irrational magnitudes. He referred to square roots as "plane irrationalities" and cubic roots as "solid irrationalities", and categorized the sums or differences of these roots, as well as the results of adding or subtracting these roots from rational magnitudes, as irrational magnitudes. He then interpreted the earlier work of Euclid using these rational and irrational magnitudes, as opposed to the geometric magnitudes used in the original text. Al-Mahani also endeavored to solve a challenge put forth by Archimedes in On the Sphere and Cylinder, Book II, Chapter 4: the division of a sphere by a plane into two volumes with a specified ratio. His efforts led to what is known in the Muslim world as "Al-Mahani's equation": x3 + c2b = cx2. The problem was deemed unsolvable until the 10th-century Persian mathematician Abu Ja'far al-Khazin successfully solved it using conic sections.

This was an outline of the multiple works done by Muslim mathematicians in the medieval era of Islam, otherwise known as Islamic Golden Age. While the details mentioned above are exhaustive in nature, developments in that era ranged from all sciences from philosophy to biology. The total amount of work is the one that survived and was translated, while most of it was lost to history during the decline of the empire. If all the work was accounted for in today's age, we would certainly have more accreditation of Muslim scientists and mathematicians for theorems and laws used today. However, as we have seen the great heights of sciences in the Golden Era of Islam, an almost sharp decline is seen today in the pursuit of this subject. The current trends and demographics of maths in the Muslim world is discussed below, along with the reasons for such trends.

Conclusion

Mathematics has been an intrinsic part of human existence, existing since the dawn of the universe and destined to persist until its end. Mathematical relationships permeate nature, reflecting the innate human qualities of curiosity, logic, and reasoning that drive exploration, discovery, and the formulation of theorems and laws we rely on today. Throughout history, humanity has utilized mathematics to provide explanations to encountered questions and solutions to faced challenges. Muslim mathematicians played a important role in advancing various branches of mathematics, notably algebra, geometry, astronomy, trigonometry and number theory. Visionaries like Al-Khwarizmi, Abu Kamil, Al-Tusi, Sharaf al-Din, Omar Khayyam, Thabit Ibn Qurra, Al-Haytham, and Abu Sahl Al-Kuhi made groundbreaking contributions that reshaped mathematical knowledge. Inspired by Islamic beliefs and teachings, these scholars emphasized the pursuit of beneficial knowledge, aligning their work with the Quran and the teachings of Prophet Muhammad. Their innovative methods for solving equations, developing geometric proofs, and applying mathematical principles to real-world scenarios laid the groundwork for modern mathematics. Beyond mathematics, their influence extended to fields like architecture, navigation, and astronomy, shaping the trajectory of scientific and intellectual progress. The legacy of these Muslim mathematicians continues to resonate, underscoring the enduring impact of their contributions on the evolution of human knowledge and understanding.

Bibliography

"Abu Sahl Waijan ibn Rustam al-Quhi." Science and Its Times: Understanding the Social Significance of Scientific Discovery. Encyclopedia.com. Accessed May 15, 2024. https://www.encyclopedia.com/science/encyclopedias-almanacs-transcripts-and-maps/abu-sahl-waijan-ibn-rustam-al-quhi.

"Al-Samaw'al, Ibn Yahya Al-Maghribi." Complete Dictionary of Scientific Biography. Encyclopedia.com. Accessed May 15, 2024.

"Al-Tusi, Sharaf Al-Din Al-Muzaffar Ibn Muhammad Ibn Al-Muzaffar." Complete Dictionary of Scientific Biography. Encyclopedia.com. Accessed May 15, 2024. https://www.encyclopedia.com/science/dictionaries-thesauruses-pictures-and-press-releases/al-tusi-sharaf-al-din-al-muzaffar-ibn-muhammad-ibn-al-muzaffar.

Abattouy, Mohammed. "Muhammad Al-Karaji: A Mathematician Engineer from the Early 11th Century." Published in June 2009. Accessed May 15, 2024. https://muslimheritage.com/muhammad-al-karaji-mathematician-engineer/.

Amir-Moez, A. R. "Khayyam's Solution of Cubic Equations." Mathematics Magazine 35, no. 5 (1962): 269-271.

Amir-Moez, A. R. "Khayyam's Solution of Cubic Equations." Mathematics Magazine 35, no. 5 (1962): 269-271.

An-Nasa'i and Al-Hakim.

Benzaari, Adam. "Number Theory in the Medieval Islamic World: The Works of Ibn al-Haytham, Omar Khayyam, and Ibn al-Banna al-Marrakushi." McGill University, Winter 2023, Department of

Mathematics and Statistics, MATH377, Jonathan Love. Accessed May 25, 2024. https://jonathanlove.info/teaching/23-NT/benzaari.pdf.

Dunham, W. "Number Theory." Encyclopedia Britannica. April 4, 2024. https://www.britannica.com/science/number-theory.

Hockey, Thomas, et al., eds. The Biographical Encyclopedia of Astronomers. New York: Springer, 2007.

Hosch, William, ed. The Britannica Guide to Algebra and Trigonometry. Britannica Educational Publishing, 2010.

Karpinski, Louis Charles. "The Algebra of Abu Kamil." The American Mathematical Monthly 21, no. 2 (1914): 37-48.

Kent, Deborah A., and David J. Muraki. "A Geometric Solution of a Cubic by Omar Khayyam… in Which Colored Diagrams Are Used Instead of Letters for the Greater Ease of Learners." The American Mathematical Monthly 123, no. 2 (2016): 149-160.

Lorch, R. "Ibn al-Haytham." Encyclopedia Britannica. February 12, 2024. https://www.britannica.com/biography/Ibn-al-Haytham.

Maor, E., and Raymond Walter Barnard. "Trigonometry." Encyclopedia Britannica. May 6, 2024. https://www.britannica.com/science/trigonometry.

Qur'an 16:12.

Qur'an 16:15-16.

Qur'an 6:97.

Qur'an 7:54.

Riyad as-Salihin, Book 13, Hadith 1391.

Sesiano, Jacques. An Introduction to the History of Algebra: Solving Equations from Mesopotamian Times to the Renaissance. Vol. 27. American Mathematical Society, 2009.

Shloming, Robert. "Historically Speaking: Thabit Qurra and the Pythagorean Theorem." The Mathematics Teacher 63, no. 6 (1970): 519-528.

Sunan Ibn Majah, 3843.

Van der Waerden, Bartel L. A History of Algebra: From al-Khwārizmī to Emmy Noether. Springer Science & Business Media, 2013.